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DEPENDENCE OF THE STARTING CHARACTERISTICS OF AN ELECTRONIC CYCLOTRON GENERATOR ON THE DISTRIBUTION OF THE HIGH FREQUENCY FIELD IN THE DIRECTION OF PARTICLE DRIFT

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ABSTRACT. Methods from the theory of weak signals are used to solve the self-consistent problem of the interaction of a spiralized electron stream with an electromagnetic field of a resonator of considerable volume, disturbing the movement of the particles, in the cases of homogeneous, gaussian and sinusoidal change of signal amplitude along the tube. A comparative analysis of the starting parameters of the corresponding types of cyclotron generators is performed; it is shown that self-excitation of the system is hindered with increasing power of the heterogeneity in the distribution of amplitudes of the hf field.

In selecting the form of oscillations to be used in a generator which /1003[1] makes use of cyclotron resonance, we always keep in mind the fact that in the area of high phase velocities of the traveling components of a standing wave in the direction of the drift of the electrons, the Doppler frequency almost corresponds to the so-called laboratory frequency. The conditions of resonance are not disrupted over a wide interval of drift velocities, i.e. the demands for homogeneity of drift velocities are reduced.

The high values of phase velocities of the wave assume small variations in the field along the tube, i.e. the usage of oscillations with minimal longitudinal index. The large number of actual amplitude distributions used in practice can be reduced to three main models: 1) the homogeneous field in which the operating "plateau" of the guiding constant magnetic field is located in the central portion of the resonator (the electron beam and collector may be outside the resonator in a relatively weak magnetic field in this case); 2) gaussian distribution, with electron movement in a quasi-optical resonator from caustic to caustic; 3) the sinusoidal law for closed wave guide resonators.

The purpose of this work is the investigation and mutual comparison of the starting characteristics of these three types of systems. As an initial system, we use the system of equations of movement and excitation formulated in [1]:

[1] Numbers in the margin indicate pagination in the foreign text.

$$\left.\begin{aligned}
\frac{d\rho}{dT} &= Af(T)\,\rho^{n-1}\cos\Theta \\
\frac{d\Theta}{dT} &= -nAf(T)\,\rho^{n-2}\sin\Theta + \Phi + \mu(\rho^2 - 1) \\
\frac{d\eta_a}{dT} &= -\frac{Af(T)}{\pi}\int_0^{2\pi}\rho^n\cos\Theta\, d\omega t_0 \\
\frac{d\eta_r}{dT} &= \frac{Af(T)}{\pi}\int_0^{2\pi}\rho^n\sin\Theta\, d\omega t_0
\end{aligned}\right\}, \qquad (1)$$

$$\rho(0) = 1,\ \Theta(0) = \omega t_0 \in [0,\ 2\pi),\ \eta_a(0) = \eta_r(0) = 0, \qquad \text{/1004}$$

where ρ is the current angular velocity, related to its initial value; $\Theta = \omega t_0 + (\omega - n\Omega)(TL/v_{\parallel}) - n\phi$ is the effective phase of the field forces acting on the electrons; ω is the frequency of the field; t_0 is the moment of entry of the electron into the interaction space; $\Omega \equiv (|e|/m_0)B_0(1 - [\beta_0^2/2])$ is the initial value of the cyclotron frequency considering the weakly relativistic correction; n is the resonance number of the cyclotron harmonic ($\omega \approx n\Omega$(; $T \equiv (z/L)$ [0,1) is the integration variable; z is the longitudinal coordinate; L is the length of the interaction space; ϕ is the current correction for the unperturbed rotation angle $\Omega(TL/v_{\parallel})$ of the electron rotating in a magnetic field with induction B_0; $v_=$ is the same thing for all electrons which do not change their drift velocity within the framework of the weakly relativistic approximation;

$$A = \frac{n^n\beta_{\perp}^{n-2}}{2^n n!}\,\frac{|e|LE_0}{m_0 c v_{\parallel}}\,|\psi^{(n-1)}(\Gamma Y)|\left(\frac{\Gamma}{k}\right)^{n-1}$$

is the dimensionless amplitude of the electrical hf field intensity, $k = \omega/c$, c is the speed of light, $\beta_{\perp} = v_{\perp}/c$, $v_{\perp}$ is the initial value of the orbital velocity of the electron; $\psi(\Gamma\eta)$ is the transverse distribution function of the electrical field; Γ is the dimensionality constant of the inverse length; Y is the coordinate of the axis of the electron spiral, $\Phi \equiv (\omega - n\Omega)(L/v_{\parallel})$ is the complete, unperturbed flight angle of a particle; $\mu \equiv (n\Omega\beta_{\perp}^2 L/2v_{\parallel})$ is the parameter of nonisochronism, determining the effectiveness of the azimuthal grouping of electrons resulting from the dependence of the cyclotron frequency on the energy of their orbital movement; $\eta(T) = \eta_a + j\eta_r = (P/I_0 U_{\perp})$ is the

current complex transverse efficiency; P is the complex power of the interaction; I_0 is the current of the beam; $U_\perp = (v_\perp^2 m_0/2|e|)$. We will use f(T) to represent the longitudinal distribution function of the amplitudes of the hf field:

$$f(T) = \begin{cases} 1 & \text{homogeneous field,} \\ e^{-(2T-1)^2} & \text{gaussian distribution,} \\ \sin \pi T & \text{sinusoidal law.} \end{cases} \qquad (2)$$

Linearizing (1) with respect to the high frequency corrections to the unperturbed values of ρ and Θ and assuming $A \ll 1$ (starting mode), then substituting the result of the solution of the first two equations of the system into the last two, we produce:

$$\left.\begin{aligned} \eta_a(L) &= nA^2 \int_0^1 f(T)\,dT \int_0^T \left\{\frac{\mu}{n} \int_0^x f(y) \sin[(T-y)\Phi]\,dy - \right. \\ &\qquad \left. - f(x)\cos[(T-x)\Phi]\right\} dx \\ \eta_r(L) &= nA^2 \int_0^1 f(T)\,dT \int_0^T \left\{\frac{\mu}{n} \int_0^x f(y)\cos[(T-y)\Phi]\,dy + \right. \\ &\qquad \left. + f(x)\sin[(T-x)\Phi]\right\} dx \end{aligned}\right\}. \qquad (3)$$

In the case of the homogeneous and sinusoidal fields, calculation of the integrals in (3) is not difficult. For the gaussian distribution, numerical solution methods were used. /1006

The results of calculations using formulas (3) for a number of values of the parameter of nonisochronism μ are presented on Figures 1-3 for the main working zone (corresponding to minimal starting currents) of self-excitation of the system. The physical sense of the quantities $nA^2/\eta_\perp$ and $\gamma = \eta_r(L)/\eta_\perp$, where $\eta_\perp \equiv \eta_a(L)$, can be clarified in the process of analysis of the balance of the complex power in the device. The first quantity is proportional to the starting current I_{st}, the second to the "hot" detuning of the generated frequency relative to the natural frequency of the resonator.

$$\left.\begin{aligned}\frac{nA^2}{\eta_\perp} &= n^3 I_{s_l} U_\perp^{n-1}\left[\frac{|\psi^{(n-1)}(\Gamma Y)|}{(n-1)}\left(\frac{n\Gamma}{2k}\right)^{n-1}\right]^2 \frac{\left|\frac{e}{m_0}\right|^n \sqrt{\mu_0/\varepsilon_0}}{2^{4-n}\pi c^{2n}} \frac{\lambda^3 Q_l}{Vk_V} N^2 \\ \gamma &= 2Q_l \frac{\omega-\omega_0}{\omega}\end{aligned}\right\}, \tag{4}$$

where Q_l is the loaded quality of the resonator; Vk_V is the effective volume occupied by the hf field in the resonator; $k_V \equiv \frac{1}{V}\int\int\int_{(V)} (E^2/E_0^2)dV$ is the usage factor of the volume; $N = (1/2\pi)\Omega L/v_\parallel$ is the number of turns in the electron spiral.

Obviously, under the condition

$$\left|\frac{\gamma\omega_0 L}{2Q_l v_\parallel}\right| \ll 1. \tag{5}$$

in the calculations for the starting characteristics we can approximately assume

$$\Phi = (\omega_0 - n\Omega)\frac{L}{v_\parallel}. \tag{6}$$

The dependence $nA^2/\eta_\perp = nA^2/\eta_\perp(\Phi)$ allows us to estimate the range of values of Φ (and also the permissible fluctuation in velocities and heterogeneity of the magnetic field) within which I_{st} increases by not more than a given quantity, for example being doubled in comparison to the minimum possible value. If we have data on the boundary values of Φ (to the left and right of the optimal value), using the graphs of $\gamma = \gamma(\Phi)$ we can then, within the framework of the ratio of the actual starting current to the minimum possible current, estimate the value of the instability of the self-excitation frequency

$$\gamma_{max} - \gamma_{min} = \frac{2Q_l}{\omega_0}(\omega_{max} - \omega_{min}). \tag{7}$$

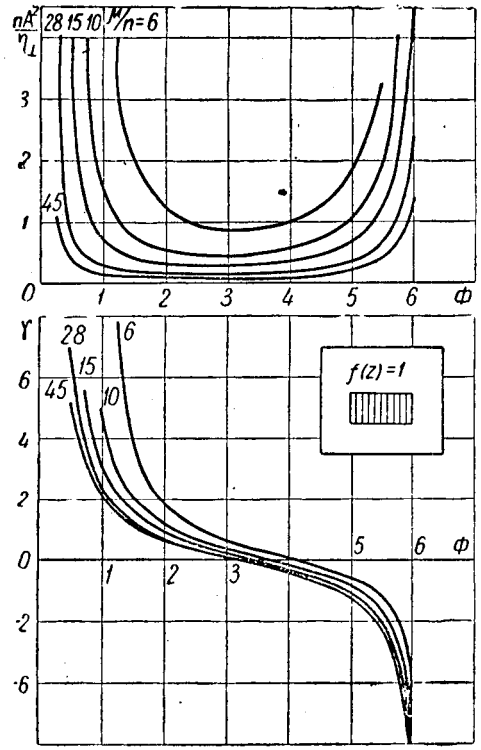

Figure 1

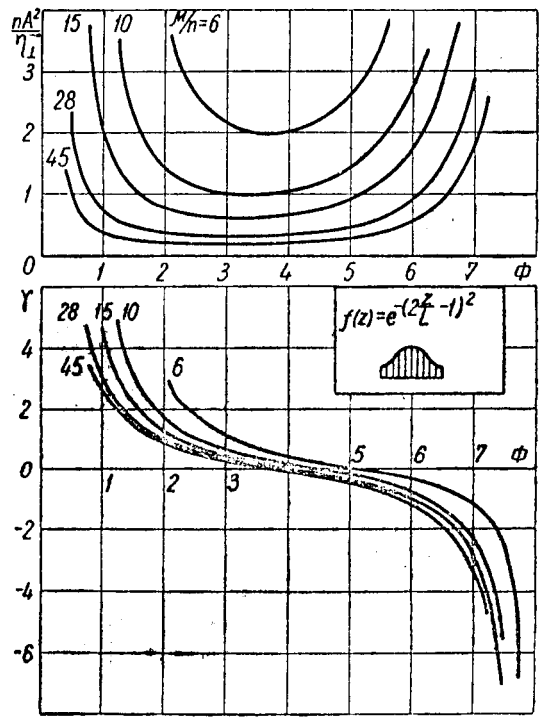

Figure 2

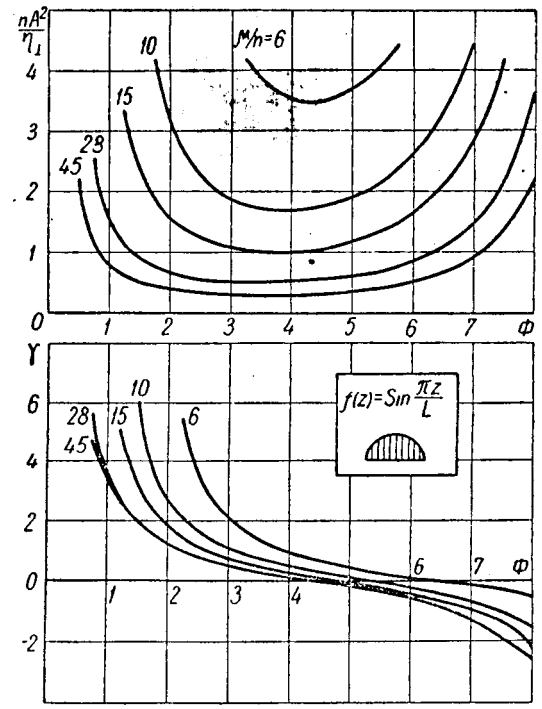

Figure 3

Investigating curves 1-3, we can conclude that of the three types of longitudinal field distributions with identical values of effective volume and quality, the easiest conditions for self-excitation correspond to the homogeneous field; as the degree by which the intensity of the hf forces in the central portion of the space of interaction increases, the achievement of the starting current values becomes continually more difficult. At the same time, /1007
the range of variations in path angle Φ determined from the condition of doubling of the starting current defined above, is broadest for the sinusoidal case. This fact is explained as a result of the simultaneous action of the stream of electrons with the two components of the standing wave moving in opposite directions, the Doppler frequencies and corresponding path angles of which differ relatively little from each other.

As an example of the usage of the results produced, let us present the calculated starting characteristics of three variants of a cyclotron generator differing in the form of function f(T), similar to the generator described in [2], at the second harmonic (n = 2) of the cyclotron frequency with oscillations of the form TE_{021} in a circular cylindrical resonator. Suppose the resonator radius r = 1.3 cm, its length L = 8 cm, the quality Q_l = 8000, the wavelength λ = 1.16 cm, $U_\perp$ = 15 kv, $U_\parallel$ = 3 kv. For our calculations according to formula (4), we assume $\psi(\Gamma Y) = J_1(K_{cr}Y)/J_{1\ max}$, $\Gamma = k_{cr} \approx k = 2\pi/\lambda$, $k_{cr}Y = 3.515$, $|\psi'(3.515)| = |\psi^*|_{max} = 0.72$:

$$k_v = \begin{cases} 0.266 & \text{homogeneous field,} \\ 0.16 & \text{gaussian distribution,} \\ 0.133 & \text{sinusoidal law.} \end{cases}$$

Based on the method outlined above, using the data of Figures 1-3 we can determine the starting values of $(nA^2/\eta_\perp)_{st\ min}$, $I_{st\ min}$, Φ_{st} and the half range $\Delta\Phi_{st}$ and correspondingly $\Delta\gamma_{st}$ (upper values in Table 1) and the corresponding values of the variation in total energies $\Delta U/U$ under the condition $v_\parallel/v_\perp$ = const for the parameters indicated above. In our calculation we did not consider the dependence of Φ on U through $v_\parallel$, which led to a certain reduction in the value of $\Delta U/U$. At the same time, we can determine the permissible fluctuation in inclination angles of the electron spirals $\Delta(\arctan v_\parallel/v_\perp)$ under the condition U = const, the permissible heterogeneity /1008
of the magnetic field $\Delta B/B$ where U = const, arc tan $v_\parallel/v_\perp$ = const and with instability of the frequency $\Delta f/f$.

TABLE 1

$f(T)$	$\left(\frac{nA^2}{\eta_\perp}\right)_{st\min}$	$\left(\frac{nA^2}{\eta_\perp}\right)_{st}$	$I_{st\min}$	I_{st}	Φ_{st}	$\Delta\Phi_{st}$	$\Delta\gamma_{st}$	$\frac{\Delta U}{U}$ %	$\Delta\left(\operatorname{arctg}\frac{v_\parallel}{v_\perp}\right)^0$	$\frac{\Delta B}{B}$ %	$\frac{\Delta f}{f}$ %
1	0,44	$\frac{0,88}{1,64}$	56	$\frac{112}{130}$	3	$\frac{1,65}{2,25}$	$\frac{1,6}{3,3}$	$\frac{10,7}{14,5}$	$\frac{14,2}{20}$	$\frac{0,41}{0,57}$	$\frac{0,01}{0,021}$
$e^{-(2T-1)^2}$	0,82	$\frac{1,64}{1,64}$	65	$\frac{130}{130}$	3,4	$\frac{1,95}{1,95}$	$\frac{1,22}{1,22}$	$\frac{12,5}{12,5}$	$\frac{14,5}{14,5}$	$\frac{0,48}{0,48}$	$\frac{0,008}{0,008}$
$\sin \pi T$	1,4	$\frac{2,8}{1,64}$	91	$\frac{182}{130}$	4	$\frac{2,33}{1,15}$	$\frac{1,1}{1,2}$	$\frac{15}{7,5}$	$\frac{15}{8}$	$\frac{0,6}{0,29}$	$\frac{0,007}{0,007}$

For comparison, we also calculated the corresponding values in the case of identical values of current $I_{st} = 2I_{st\ min}$ for all three variants of f(T), where $I_{st\ min}$ is the starting current of the idealized instrument with gaussian field distribution (lower values in Table 1).

We can see that of the three variants of the longitudinal distribution of the field, the most stable in relation to variations in kinematic characteristics of electron movement is the instrument with the sinusoidal field, which also has the greatest frequency stability. Nevertheless, provision of the required homogeneity of the components of the velocity vector and the focusing field can be successfully realized for all of the systems analyzed.

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